**Fluid Statics**

**Application of the Linear Momentum Principle**

**Developing a Relationship for Pressure Variation in a Static Fluid**

General momentum principle application on a static fluid will be utilized to develop a relationship that describes the pressure variation in the fluid. The steps are as described below.

Consider a large reservoir of fluid kept stationary. The distance of the surface of the fluid from the bottom of a tank is L. Assume the height of the tank is in the z direction. Z = 0 is the bottom of the tank and z = L is the surface of the liquid. The goal is to find a general expression for pressure at any depth “h” from the surface of the liquid.

**Step 1**: Consider a differential element of fluid at z in the fluid with dimensions Δx, Δy, and Δz. Write the momentum balance equation for this element.

Differential volume (ΔV)

H = L - z

Static fluid: 0 = sum of body forces + sum of surface forces

0 = ⍴**g**ΔV + **i**pxΔyΔz - **i**px+ΔxΔyΔz + **j**pyΔxΔz - **j**py+ΔyΔxΔz + **k**pzΔxΔy - **k**pz+zΔxΔy

0 = ⍴**g**ΔxΔyΔz + **i**pxΔyΔz - **i**px+ΔxΔyΔz + **j**pyΔxΔz - **j**py+ΔyΔxΔz + **k**pzΔxΔy - **k**pz+zΔxΔy

**Step 2:** Divide by ΔxΔyΔz and take limits Δx -> 0, Δy -> 0, and Δz -> 0.

0 = -⍴**k**g + (**i**(px - px+Δx))/Δx + ((**j**(py - py+Δy))/Δy + ((**i**(pz - pz+Δz))/Δz

0 = -⍴**k**g - [**i** δP/δx + **j** δP/δy + **k** δP/δz]

**Step 3:** Equate specific terms to 0.

Static: **i**, **j**, **k** orthogonal can only be static if ∑ **i** = 0, ∑ **j** = 0, ∑ **k** = 0

δP/δx = 0, δP/δy = 0, δP/δz = - ⍴g

**Step 4:** Integrate the pressure variation equation with the proper boundary condition.

δP/δz = - ⍴g

Integrate between limits to get pressure variation in z direction.

Within small differential limits, phase change pressures do not change.

Identify boundary conditions

Z = L, p = Patm [from continuous property of pressure variation] at integral, P changes as a continuous function

δP/δz = - ⍴g

P = - ⍴gz + ci

Since at z = L, p = Patm

Patm = - ⍴gL + ci

Ci = Patm + ⍴gL

P = - ⍴gz + Patm + ⍴gL = Patm + ⍴g(L-z)

H = (L-z)

P = Patm + ⍴gh